Physics-Constrained and Vision-Informed Friction Coefficient Estimation

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***Abstract*— Estimation of tire surface friction coefficients when moving over diverse and changing terrain is challenging. Estimation of friction coefficients is critical to ensuring that model-based control approaches can generate controls that track a desired reference trajectory, and maintain passenger safety. Utilizing a combined physics-constrained and vision-informed approach can identify shifting terrain, and quickly fit friction estimates that enable performant control. We use gradient-based optimization in conjunction with dynamics models to calculate coefficient estimates, and vision algorithms to identify the current terrain. This approach enables model-based control approaches in off-road and adverse environments without expensive system identification, keeping people and equipment safe.**

I. Introduction

We present an approach to identify tire-surface friction coefficients in settings with changing road surfaces. Estimation of tire-surface friction coefficients is difficult, dependent upon difficult to estimate true friction coefficients and state information. Approaches that rely on directly sensing or estimating the friction of the surface are subject to error due to surfaces that may appear the same to a suite of sensors, but have differing friction values [1]. Approaches that are purely data-driven are dependent on the completeness and accuracy of the dataset. Dynamics-based approaches use dynamics models and filtering approaches to obtain road-surface friction estimates.

We present a fusion approach that uses visual information to cluster surfaces into groups, and then uses a dynamics model paired with a gradient-based solver to identify model parameters related to tire-surface friction. Our approach uniquely pairs surface clustering and a distributional approach to friction estimation driven by physics-constrained optimization, for multi-surface off-road friction estimation.

II. Problem Statement

The tire-surface friction parameters we seek to identify are present in our dynamics model as a Pacjeka tire model [2]. We observe a series of states $x=(s,i)\in S\times I$, where S is the space of state variables, and I is the space of images. We also record the actions applied to the system $a\in A$. We receive one observation for each timestep in the interval $0\leq t\leq T$. To model the dynamics of the vehicle, we have a model $\dot{s}=f(s,a|\theta_d,\theta_f)$, which governs the evolution of the state s, subject to the current state and action, and two sets of parameters, θ_f which is the set of tire-surface friction parameters, and θ_d, which is the set of all other parameters. We assume that the parameters θ_d are known apriori and are time invariant.

We group terrain into clusters c_i based on the state. For each cluster, we have a distribution of friction values $\theta_f^i\sim\mathcal{N}(\mu_f^i,\sigma_f^i)$, which parametrize a normal distribution. The full set of friction distributions is $\Theta_f=\{\theta_f^1,...,\theta_f^C\}$, which we aim to find. For each timestep, the correct cluster is identified, and a specific value $\hat{\theta}_f$ is drawn from the relevant distribution. We use the symbol $\sim$ to denote that a sample is drawn from a distribution. We formulate this as an optimization problem as follows:

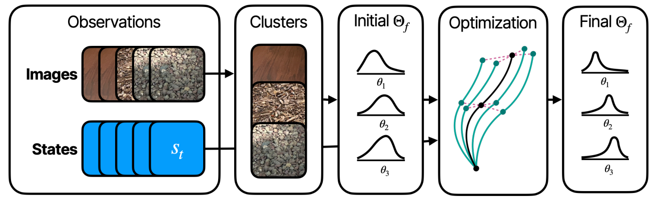

Fig. 1. This figure shows the proposed approach, detailed in Equation (2). Images from our observation are clustered, and an initial guess and the states observed are used to optimize the parameter estimates.

$$\Theta_f = \underset{\hat{\Theta}_f,\hat{c}_t}{\text{minimize}} \quad \mathbb{E}_{\Theta_f}\left[\sum_{t=0}^{T}\|s_t-\hat{s}_t\|^2\right] \tag{1a}$$

$$\text{subject to} \quad \hat{s}_0=s_0, \tag{1b}$$

$$\hat{c}_t=\text{Cluster}(s_i), \tag{1c}$$

$$\hat{\theta}_f\sim\theta_f^{\hat{c}_t}, \tag{1d}$$

$$\hat{s}_{t+1}=f(s_t,a_t|\theta_d,\hat{\theta}_f) \tag{1e}$$

In (1a), we minimize the L2 norm of the error between our predicted states $\hat{s}$ given Θ_f and the observed state s. The first constraint (1b) ensures the initial state matches the observed state. The second constraint (1c) maps each state to a terrain cluster. The third constraint (1d) maps our current estimate of the parameters $\hat{\theta}_f$ to a parametrized distribution $\theta_f^{\hat{c}_t}$. Finally, we update the state using our dynamics and the parameters corresponding to the current terrain cluster in (1e).

III. Methods

We propose to solve this optimization problem (1) by breaking it down into a dual minimization, specified in Equation (2).The original problem is then solved in two steps: first, identify the terrain cluster that the vehicle is currently on by feeding the current state and input image to our vision-informed friction clustering (VFC), and then minimize the parameters of our set $\hat{\Theta}_f$ using a physics-informed optimizer. An overview of this architecture is in Figure 1.

The VFC is based on a ResNet [3] backbone, and trained to generate image embeddings that are used to create our terrain clusters c_t. We then apply an off-the-shelf clustering algorithm, like Mean Shift [4]. During the optimization process, we find the closest point for which we have an image, and use the associated terrain cluster.

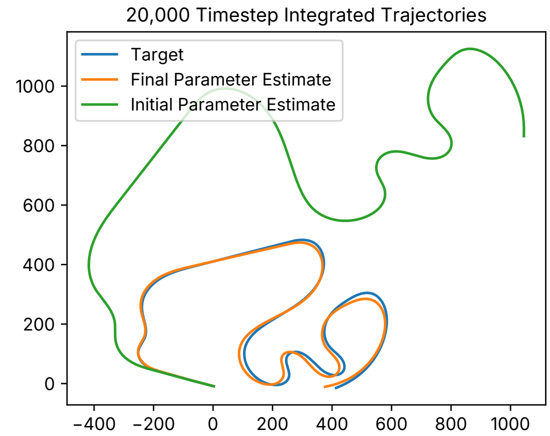

Fig. 2. Numerical simulation results that show the improvement in the parameter estimate. The target data was generated with a multi-body model, and the parameter estimates are generated with a dynamic bicycle model with Pajecka tire forces.

$$\min_{\hat{\Theta}_f} \quad \min_{\hat{c}_t} \quad \mathbb{E}_{\Theta_f}\left[\sum_{t=0}^{T} \|s_t - \hat{s}_t\|^2\right] \tag{2a}$$

$$\text{subject to} \quad \hat{c}_t = \text{VFC}(s_t, i_t), \tag{2b}$$

$$\hat{\theta}_f \sim \theta_f^{\hat{c}_t}, \tag{2c}$$

$$\hat{s}_0 = s_0, \tag{2d}$$

$$\hat{s}_{t+1} = f(s_t, a_t | \theta_d, \hat{\theta}_f) \tag{2e}$$

We create a set of friction parameters θ_f^t for each of the clusters, and optimize by exploiting differentiable dynamics. We integrate the ODE with the current set of parameters, and use PyTorch's auto differentiation to calculate gradients and optimize the friction parameters [5]. To better identify the distribution that the parameters belong to, we draw multiple values from the distributions Θ_f, solving this problem in a multiple-shooting fashion.

IV. Results

We plan to evaluate our approach in three settings. First, a numerical simulation environment, where the plant model is a multi-body dynamics model [6] following a reference trajectory on changing terrain, where the exact friction coefficients and surfaces are known. Second, a photorealistic simulation environment, CARLA [7], where the vehicle follows a reference trajectory on changing terrain, including ice, asphalt, and grass. Finally, we evaluate our approach on a 1:5 scaled off-road platform, driving on grass, stone, and gravel.

A. Current Results: Numerical Simulation

We generated data using an extension of the F1Tenth Gym simulator [8], which incorporates multi-body physics. The model used in our approach was a dynamic bicycle model with Pajecka tire forces [9]. The objective of this experiment is to observe the performance of the physics-constrained optimization algorithm when the ground-truth terrain is given, and true state information is known.

Our initial parameter estimate was taken from a nominal tire force model. The plot in Figure 2 shows the recorded trajectory in blue, and trajectories generated with our dynamic bicycle model in green and orange. The green trajectory represents the initial parameter estimate, and the orange represents the final parameter estimate. The green and orange trajectories are integrated from the initial condition for 20,000 timesteps, with a lower fidelity model than that used to generate the recorded trajectory, specifically a single track dynamic bicycle model.

Fig. 3. The 1/5th scale platform we plan to use to evaluate our approach.

B. Future Experiments

The photorealistic CARLA simulator will be used to run one set of experiments, with the aim of testing the performance of the approach when true state information is given, and the terrain clustering is done via clustering of image data. We will evaluate on ice, asphalt, and grass. Our final experiment will explore the efficacy of our approach in a real off-road robotics platform, pictured in Figure 3, where our robot will traverse asphalt, grass, gravel and stone.

V. Background

Much of the existing work in tire surface friction coefficient estimation focuses on the identification of tire-road friction. There are three families of approaches to identifying tire-road friction coefficients, direct sensing, dynamics-based approaches, and data-driven approaches [1]. Direct sensing approaches use camera, laser, and other sensor data to capture the roughness of the surface [10], [11]. Dynamics-based approaches use a dynamic vehicle model and state information from the vehicle to identify tire-road friction [12], [13], [14], [15]. Learning-based approaches use historical data and machine learning to predict tire-road friction [16]. Finally, there are hybrid approaches, which combine the strengths of different methodologies to improve estimation precision and reliability. For instance, integrating camera-based road surface analysis with dynamic tire modeling can significantly enhance the accuracy of friction estimations [17], [18]. Still other approaches attempt to model the full dynamics from visual input [19]. Our approach has the advantage of not needing any labeled data as both the parameters and the clusters are learnt.

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